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SHORTER COMMUNICATIONS

TEMPERATURE DISTRIBUTION IN A COMPOSITE PRISMATIC ROD IN THE CASE OF HEAT GENERATION IN THE CIRCULAR, CYLINDRICAL CONCENTRIC CORE

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NOMENCLATURE

a_p, apothem of the polygon;
A_s, constant [see equation (7)];
k_1, thermal conductivity coefficient of the circular concentric core (Sub-domain I, see Fig. 1);
k_2, thermal conductivity coefficient (Sub-domain II, see Fig. 1);
q_1, heat flux for $R = R_1$;
R, arbitrary radius;
R_1, radius of the circular concentric core;
r_1, radius in the ξ-plane [see equation (10)];
$\dot{S}$, heat generation constant;
T, temperature;
T_0, wall temperature (outer boundary);
T_1, temperature (at $R = R_1$).

INTRODUCTION

CONSIDER the thermal system shown in Fig. 1(a) governed by the differential system:

Sub-domain I:

$$k_1\nabla^2 T = -\dot{S} \quad (0 \leqslant R \leqslant R_1) \tag{1}$$

where $\dot{S}$ is the heat generation constant.

Sub-domain II:

$$\nabla^2 T = 0 \quad (R \geqslant R_1) \tag{2}$$

$$T[L(x,y) = 0] = T_0 \tag{3}$$

where $L(x,y) = 0$ denotes the functional relation which defines the boundary of the domain.

For the sake of generality it is assumed that the material of sub-domain II is characterized, from a thermal viewpoint, by a conductivity coefficient k_2.

An exact solution of the governing differential system appears to be out of the question.

An approximate, analytical approach based on the conformal mapping technique is developed in the present paper. The results are in good agreement with values obtained by means of the finite element method.

It is important to point out that configurations such as that shown in Fig. 1 and whose thermal behavior is governed by equations (1)–(3) are of basic importance in several fields of applied sciences: biomedicine, nuclear and chemical engineering, etc.

For instance, a cylindrical rod with heat generation is usually taken as a reasonable approximation to estimate the temperature rise in a working muscle fiber. Figure 1 corresponds to a more general situation where a group of working muscle fibers is surrounded by a material of non-heat generation characteristics and with a non-circular boundary configuration.

APPROXIMATE, ANALYTICAL SOLUTION

If $R_1 \ll a_p$ it is reasonable to make the approximation:

$$T(R,\phi)/R = R_1 = T_1 \tag{4}$$

where T_1 is a constant value which will be determined at a later stage.

Accordingly, the solution of the differential system (1) and (4) is given by:

$$T = T_1 + \frac{\dot{S}}{4k_1}(R_1^2 - R^2) \tag{5}$$

and the heat flux, for $R = R_1$, results:

$$q_1 = -k_1 \frac{\mathrm{d}T}{\mathrm{d}R}\bigg/ R = R_1 = \frac{\dot{S}\cdot R_1}{2}. \tag{6}$$

In the sub-domain II the thermal problem is defined by equations (2) and (3). Since (2) is Laplace's equation one can make use of the conformal mapping method.

It has already been shown [2], that the approximate transformation of sub-domain II in the z-plane onto an annulus in the ξ-plane is an easy task if $R_1/a_p \ll 1$ [see Fig. 1(b)].

Regular, polygonal domains are mapped conformally onto a unit circle in the ξ-plane by the functional relation:

$$z = a_p\cdot A_s \sum_{j=0}^{\infty} a_{1+js}\xi^{1+js}; \quad a_1 = 1 \tag{7}$$

where s is the order of the polygon. The parameters A_s and a_{1+js} are tabulated in [2].

When $R_1/a_p \ll 1$ one has:

$$z \simeq a_p\cdot A_s\cdot \xi \tag{8}$$

and accordingly:

$$R_1 \simeq a_p\cdot A_s\cdot r_1. \tag{9}$$

Then, the radius of a circle in the ξ-plane which maps, approximately, onto a circle of radius $R = R_1$ in the z-plane is given by:

$$r_1 = R_1/a_p\cdot A_s. \tag{10}$$

Assuming an ideal thermal contact between the sub-domains I and II for $R = R_1$, the heat flux entering II is given by (6) and the total heat results:

$$q = \dot{S}\cdot\pi\cdot R_1^2. \tag{11}$$

In the ξ-plane one has:

$$q = -k_2\cdot 2\pi\cdot r\cdot\frac{\mathrm{d}T}{\mathrm{d}r}. \tag{12}$$

Accordingly:

$$q\cdot\frac{\mathrm{d}r}{r} = -k_2\cdot 2\pi\cdot \mathrm{d}T \tag{13}$$

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‡This problem is treated in Seagrave's excellent textbook [1].

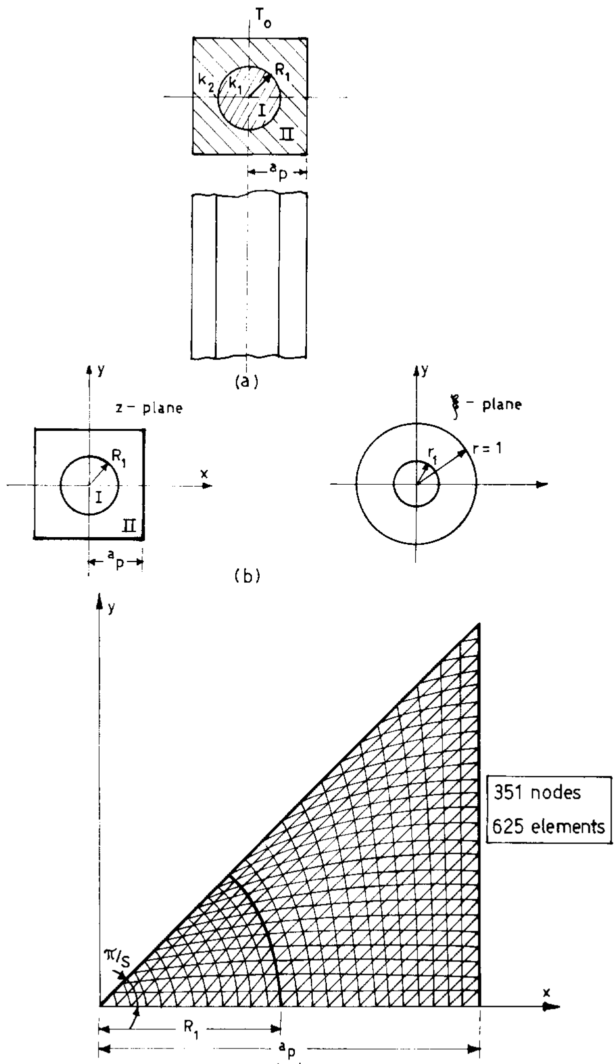

FIG. 1. (a) System under study. (b) Approximate conformal mapping of the domain. (c) Finite element distribution.

and integrating between r_1 and 1 one obtains:

$$q \cdot \ln \frac{1}{r_1} = -2\pi \cdot k_2 (T_0 - T_1). \tag{14}$$

Substituting (10) in (14) results in the relation:

$$T_1 \simeq T_0 - \frac{q}{2\pi \cdot k_2} [\ln(R_1/a_p) - \ln A_s]. \tag{15}$$

From equations (5), (11) and (15) one determines the temperature variation in the sub-domain I which in non-dimensional form results:

$$\frac{T - T_0}{\dfrac{\dot{S} \cdot R_1^2}{k_1}} = \frac{1}{4}\left[1 - \left(\frac{R}{R_1}\right)^2\right] - \frac{1}{2(k_2/k_1)}\left[\ln\left(\frac{R_1}{a_p}\right) - \ln A_s\right]. \tag{16}$$

On the other hand, the temperature distribution in the image of the sub-domain II is given by (13) integrated between r_1 and r:

$$q \cdot \ln \frac{1}{r} = 2\pi \cdot k_2 (T - T_0) \tag{17}$$

Table 1. Comparison of results in the case of a prismatic, square rod ($k_2/k_1 = 2$)

$$\frac{T - T_0}{\dot{S} \cdot R_1^2/k_1}$$

R/a_p	$R_1/a_p = 0.20;\ \phi = 0$		$R_1/a_p = 0.50;\ \phi = 0$	
	Analytical	Finite elements	Analytical	Finite elements
0	0.6713	0.6715	0.4422	0.4425
0.08	0.6313	0.6298	0.4358	0.4358
0.16	0.5113	0.5090	0.4166	0.4165
0.24	0.3757	0.3734	0.3846	0.3844
0.32	0.3036	0.3020	0.3398	0.3394
0.40	0.2475	0.2463	0.2822	0.2814
0.48	0.2014	0.2004	0.2110	0.2092
0.56	0.1621	0.1613	0.1621	0.1615
0.64	0.1274	0.1268	0.1274	0.1271
0.72	0.0961	0.0956	0.0961	0.0958
0.80	0.0671	0.0667	0.0671	0.0669
0.88	0.0397	0.0395	0.0396	0.0396
0.96	0.0134	0.0131	0.0134	0.0131
1.00	0.0012	0.0000	0.0012	0.0000

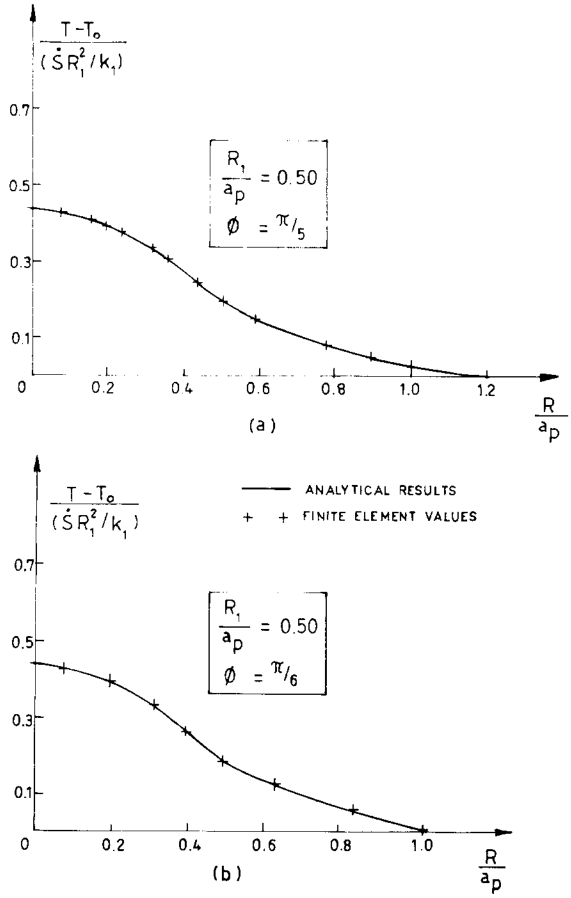

FIG. 2. Dimensionless temperature distribution. (a) Pentagonal outer boundary. (b) Hexagonal outer boundary.

and from (11) and (17) one finally obtains:

$$\frac{T-T_0}{\frac{\dot{S}\cdot R_1^2}{k_1}} = -\frac{\ln r}{2(k_2/k_1)}. \tag{18}$$

Using now equations (7) and (18) one determines the thermal field in the sub-domain II.

Numerical results of the problem governed by equations (1), (2) and (3) were also obtained using a finite element code.* The domain is subdivided into triangular elements and a linear variation of the temperature field is assumed inside the element [see Fig. 1(c)].

The accuracy of the algorithm is quite satisfactory from a practical viewpoint since it yields an agreement better than 0.5% with exact solutions when using a subdivision similar to that shown in Fig. 1(c).

COMPARISON OF RESULTS AND CONCLUSIONS

Table 1 depicts a comparison of results for an outer square shape ($R_1/R_p = 0.20$ and 0.50; $\phi = 0^\circ$).

Figure 2 deals with pentagonal and hexagonal boundaries and the dimensionless temperature parameter has been plotted for $\phi = \pi/5$ and $\pi/6$, respectively.

All calculations have been performed taking $k_2/k_1 = 2$.

It may be concluded that the agreement is remarkably good for the cases considered and even in the cases where the values of R_1/a_p are relatively large (in spite of the approximations involved when using the analytical approach).

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NATURAL CONVECTION FROM SINGLE HORIZONTAL PLATINUM WIRES

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NOMENCLATURE

- C_p, fluid heat capacity at constant pressure;
- D, wire diameter;
- g, acceleration of gravity;
- Gr, Grashof number, $l^3\rho^2 g\beta\theta/\eta^2$;
- h, heat-transfer coefficient;
- l, wire length;
- Nu, Nusselt number, hD/λ;
- Pr, Prandtl number, $C_p\eta/\lambda$;
- T, temperature;
- T_0, temperature of the enclosure.

Greek symbols

- β, fluid coefficient of thermal expansion;
- η, dynamic viscosity;
- θ, wire to wall temperature difference;
- λ, fluid thermal conductivity;
- ρ, fluid density.

INTRODUCTION

CONVECTIVE heat transfer is governed by the laws of fluid flow, the equation of continuity, and the equation for the heat flow in a moving fluid. An exact solution of these equations with particular boundary conditions is not feasible except in certain simple cases. However, important relationships may be obtained from these equations by means of the theory of similarity. Thus for a natural convection the Nusselt number should be a function of the Grashof number and the Prandtl number, i.e.

$$Nu = f(Gr, Pr). \tag{1}$$

The form of $f(Gr, Pr)$ can be determined either strictly experimentally or by using theoretical analysis with some experimental information.

Our current understanding of convective heat transfer from circular cylinders has been summarized in the recent review by Morgan [1]. According to his extensive survey